Closing Today: HW_2A, 2B, 2C
Closing Next Wed: HW_3A, 3B, 3C
Exam 1 is next Thurs (4.9, 5.1-5.5, 6.1-6.3)

Entry Task:
Using substitution, evaluate:

$$(a) \int \frac{(\ln(x))^3}{x}\, dx$$

$$(b) \int_1^2 e^{5x}\, dx$$

$$(c) \int \frac{x^5}{x^3+1} dx$$

(a) $\int \frac{u^3}{\cancel{x}} \cancel{x}\, du$

$u = \ln(x)$

$du = \frac{1}{x} dx$

$x\,du = dx$

$= \frac{1}{4} u^4 + C$

$= \boxed{\frac{1}{4}(\ln(x))^4 + C}$

CHECK!

(b) $\int_1^2 e^{5x} dx$

$u = 5x$

$du = 5 dx$

$\frac{1}{5} du = dx$

$\int_5^{10} e^u \frac{1}{5} du$

$= \frac{1}{5} e^u \Big|_5^{10}$

$= \boxed{\frac{1}{5}(e^{10} - e^5)}$

(c) $\int \frac{x^5}{u} \frac{1}{3x^2} du$

$u = x^3 + 1$

$du = 3x^2 dx$

$\frac{1}{3x^2} du = dx$

$= \frac{1}{3} \int \frac{x^3}{u} du$

$x^3 = u - 1$

$= \frac{1}{3} \int \frac{u-1}{u} du$

$= \frac{1}{3} \int 1 - \frac{1}{u} du$

$= \frac{1}{3} (u - \ln|u|) + C$

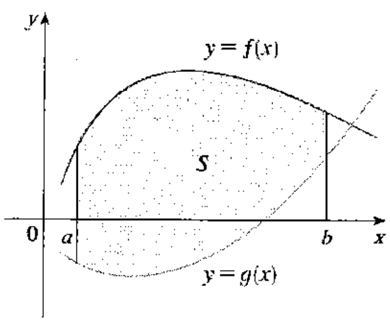
$= \boxed{\frac{1}{3}(x^3+1 - \ln|x^3+1|) + C}$

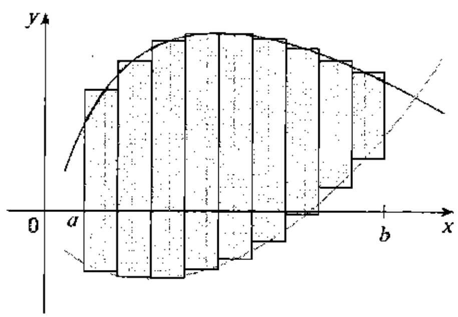
CHECK!

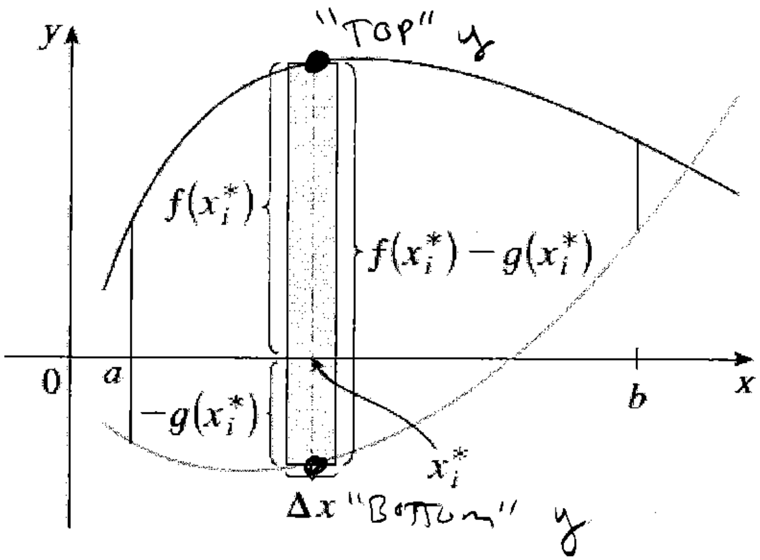
Ch 6: Basic Integral Applications

6.1 Areas Between Curves

Using dx:







(a) Typical rectangle

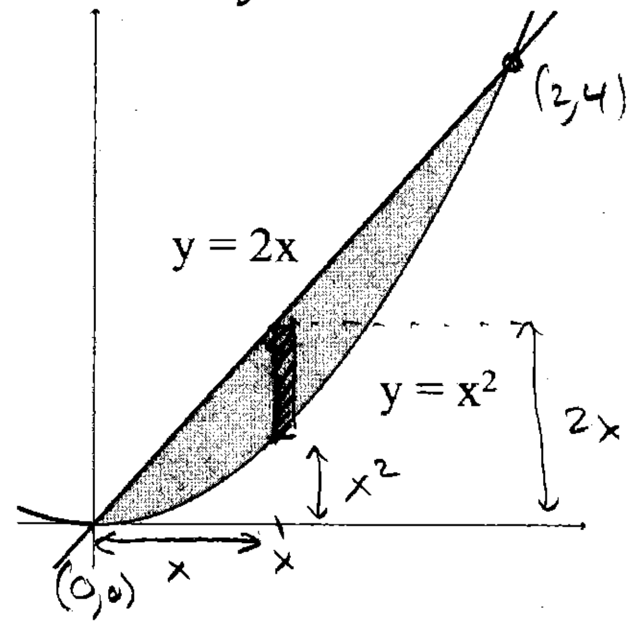
$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} (f(x_i) - g(x_i))\Delta x$$

Example: Find the area bounded between $y = 2x$ and $y = x^2$.

$2x = x^2$

$\Rightarrow 0 = x^2 - 2x$

$0 = x(x-2)$



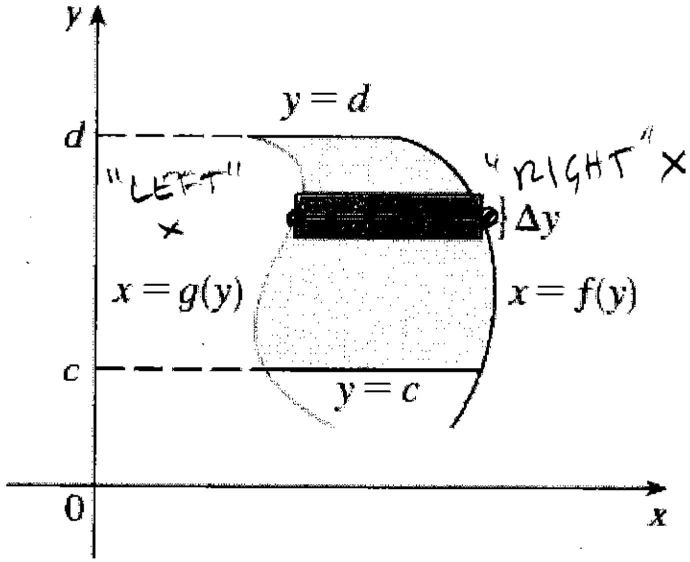
$$\int_0^2 2x - x^2 \, dx$$

$$x^2 - \frac{1}{3}x^3 \Big|_0^2 = \left((2)^2 - \frac{1}{3}(2)^3\right) - \left(0^2 - \frac{1}{3}(0)^3\right)$$

$$= 4 - \frac{8}{3}$$

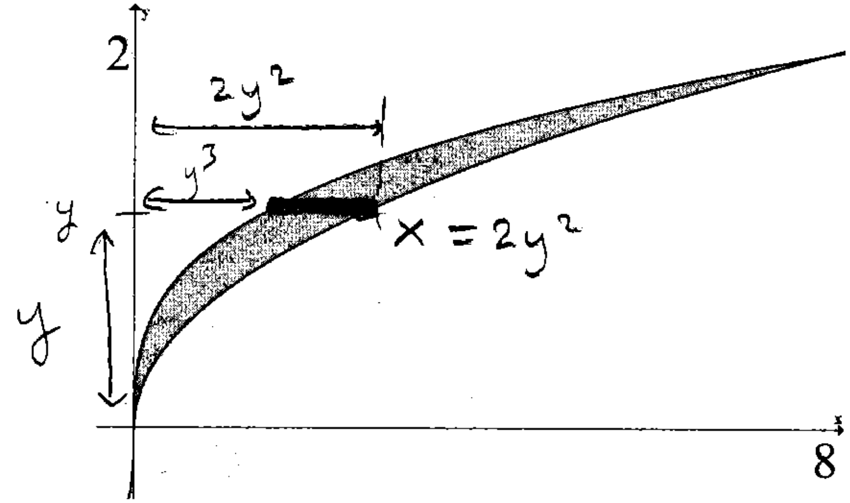
$$= \boxed{\frac{4}{3}}$$

Using dy:



$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} (f(y_i) - g(y_i))\Delta y$$

Example: Set up an integral for the area bounded between $x = 2y^2$ and $x = y^3$ (shown below) using dy.



$$\int_0^2 2y^2 - y^3 \, dy$$

$$= \frac{2}{3}y^3 - \frac{1}{4}y^4 \Big|_0^2$$

$$= \left(\frac{2}{3}(2)^3 - \frac{1}{4}(2)^4\right) - 0$$

$$= \frac{16}{3} - 4 = \boxed{4/3}$$

Summary: The area between curves

1. Draw picture finding all intersections.
2. Choose dx or dy. Get ***everything*** in terms of the variable you choose.
3. Draw a typical approx. rectangle.
4. Set up as follows:

$$\text{Area} = \int_a^b (\text{TOP} - \text{BOTTOM})\, dx$$

$$\text{Area} = \int_c^d (\text{RIGHT} - \text{LEFT})\, dy$$

HERE IS WHAT IT WOULD LOOK LIKE

$$\int_0^1 \sqrt{x} - -\sqrt{x}\,dx + \int_1^4 \sqrt{x} - (x-2)\,dx$$

ALSO CORRECT

Example: Set up an integral (or integrals) that give the area of the region bounded by x = y^2 and y = x – 2.

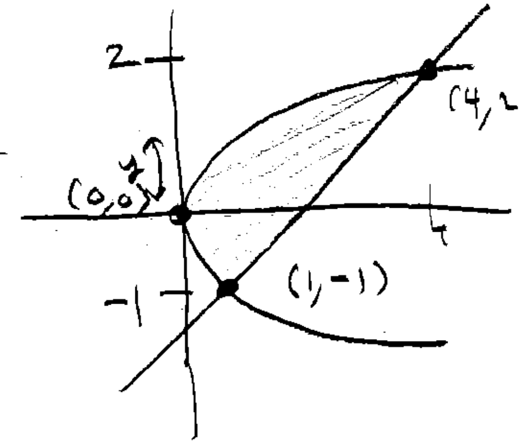
$$x = y^2 \iff \begin{cases} y = \sqrt{x} \\ y = -\sqrt{x} \end{cases}$$

$$x = y + 2 \iff y = x - 2$$

$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$



DON'T USE X! $\left\{ \begin{array}{l} \text{BOTTOM CHANGES} \\ \text{AT } x = 1 \end{array} \right.$

RIGHT − LEFT

$$\int_{-1}^{2} y + 2 - y^2\, dy$$

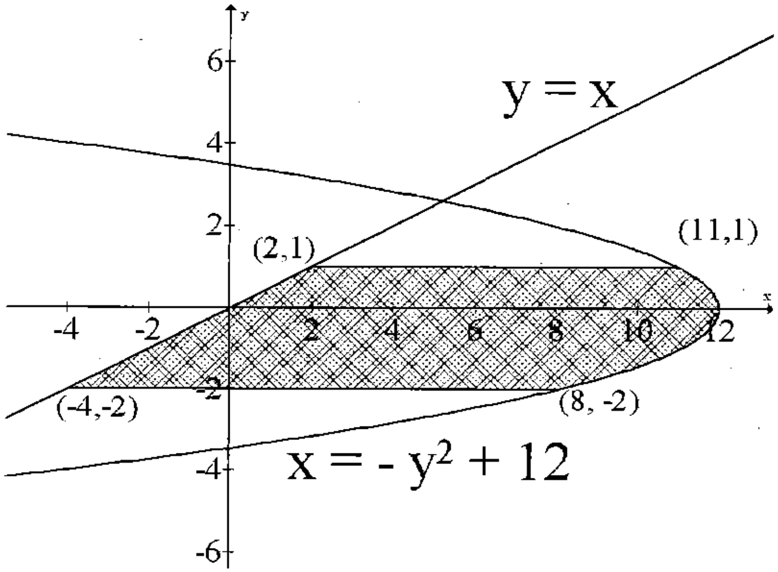
$$\frac{1}{2}y^2 + 2y - \frac{1}{3}y^3 \Big|_{-1}^{2}$$

$$\left(\frac{1}{2}(2)^2 + 2(2) - \frac{1}{3}(2)^3\right) - \left(\frac{1}{2}(-1)^2 + 2(-1) - \frac{1}{3}(-1)^3\right)$$

$$\left(2 + 4 - 8/3\right) - \left(\frac{1}{2} - 2 + 1/3\right)$$

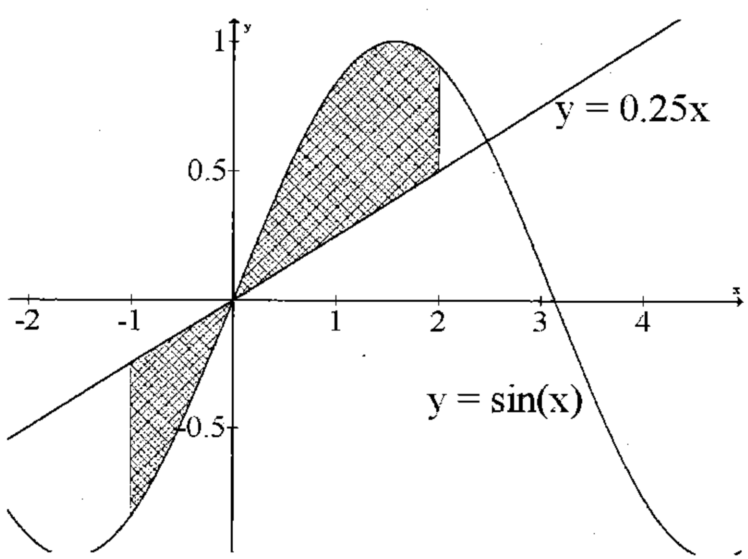
$$8 - 9/3 - 1/2 = 5 - 1/2 = \boxed{9/2}$$

Set up an integral for the total positive area of the following regions:



USE dy!!!

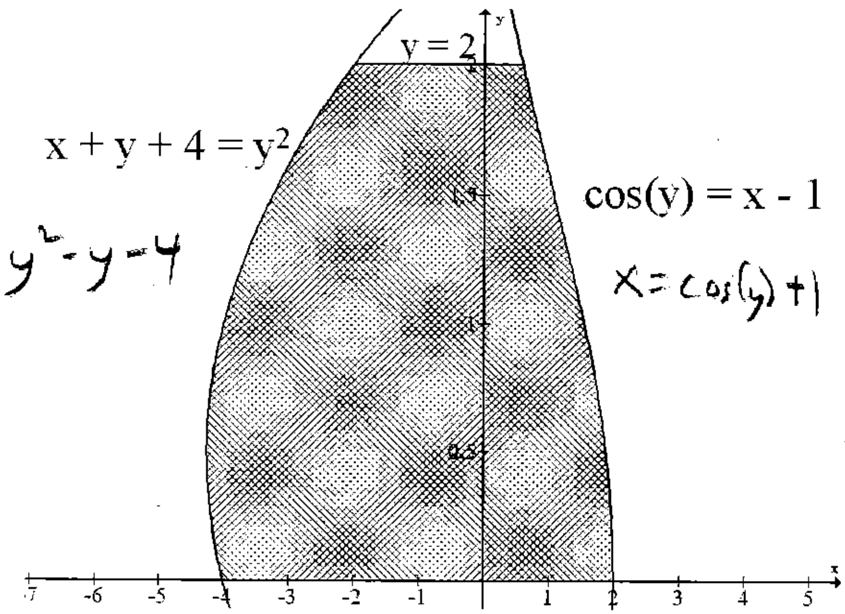
$$\int_{-2}^{2} (-y^2+12) - y \, dy$$



USE dx!!!

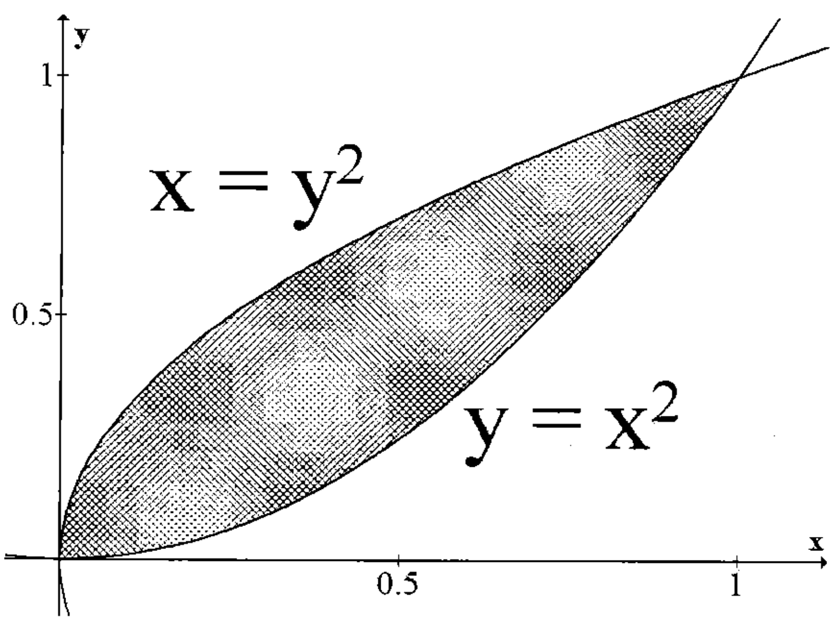
$$\int_{-1}^{0} \frac{1}{4}x - \sin(x)\, dx + \int_{0}^{2} \sin(x) - \frac{1}{4}x \, dx$$

$x = y^2 - y - 4$



USE dy!!!

$$\boxed{\int_0^2 (\cos(y)+1) - (y^2-y-4)\,dy}$$



BOTH WORK WELL

dx: $\boxed{\int_0^1 \sqrt{x} - x^2\,dx}$

$dy = \boxed{\int_0^1 \sqrt{y} - y^2\,dy}$